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Laminar diffusion of suspended particulate matter using a two phase flow model

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SUMMARY

The present paper envisages laminar mixing of a two-dimensional jet of particulate suspension in an incompressible carrier fluid with a free stream in direction of the jet axis. Finite difference technique has been employed for finding out solution of governing equations. It is found that the diffusion parameter ε, the ratio of particle diffusion coefficient and kinematic viscosity of the carrier fluid, have significant influence on the concentration of particles. A large value of ε has the effect in increasing the perturbation velocity u_{p} and perturbation density ρ_{p}. It is observed that the volume fraction φ, has no significant effect on perturbation velocity u and u_{p} but has profound effect on perturbation velocity v and v_{p}. It is also found that the particle phase as well as the carrier fluid velocity attain free stream value for the large ξ, the modified x-co-ordinate. Further the magnitude of the perturbation quantities $u, u_{\mathrm{p}}, v, v_{\mathrm{p}}$ decreases as ξ increases i.e. at far away from the nozzle exit. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: particulate suspension; boundary layer characteristics; finite difference techniques; volume fraction

1. INTRODUCTION

Now-a-days, when increasing environmental pollution is a world-wide problem, the need of controlling the pollution is vital. Modelling air and water pollution reveals awareness and serves as an important tool for environmental protection and to abate, to control and to prevent degradation. A good model is suitable for various temporal and spatial scales because it is physically realistic, accurate and universal. Up till now closed form of analytical modellings available are (1) Gaussian plume model, (2) Gaussian puff model, (3) first-order closure model, (4) Eulerian Grid model, (5) Lagrangian trajectory model, (6) particle-in cell model, and (7) Random walk trajectory particle model. In the above-mentioned models, dispersions of pollutant through the ambient fluid are studied. Soo [1], Marble [2], Singleton [3], Saffman [4],

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Batchelor [14] and Hinze [15] have studied various aspects of two-phase flow by neglecting volume fraction and diffusion of particles through carrier fluid. Outterman [5] has studied particle migration of laminar mixing of a suspension with a clear fluid where the volume fraction and diffusion of particles have been neglected. Ryhming [6] has decided upon a simplest, yet relevant model of a two-phase jet flow where Stoke's drag force and particle diffusion through the fluid phase are taken into consideration. Dutta and Mishra [7] have shown the effect of transverse force on laminar mixing of a two-dimensional jet of particulate suspension with a moving freestream on the basis of the model proposed by Ryhming [6]. Dutta and Das [8] have studied the laminar jet mixing of a fluid with SPM issuing from a circular orifice. The diffusion of pollutants in the form of SPM of different sizes, and density much higher than the carrier fluid, can very well be studied by considering the model suggested by Rhyming [6]. In the present study the effect of lift-force due to Brownian motion and volume fraction of SPM are considered in addition to study the effect of drag force and particle diffusion on the laminar mixing of a two-dimensional jet with the moving free stream in the direction of the jet axis.

2. MATHEMATICAL FORMULATION OF THE MODEL

In the available mathematical analysis of flows with SPM the volume fraction (the volume occupied by the SPM) is neglected when,

(i) the particles represent less than one-half of the mass of the mixture.
(ii) density of the particle material is more than a thousand times larger than the gas density. In such cases the volume fraction is of order 10^4 and the assumption of negligible volume fraction is well justified. But at the high gas densities (high pressure) and at high particle mass fraction, the volume fraction of the particles, will be sufficiently large so that the volume fraction should not have been neglected as suggested by Rudinger [9].

The present problem under study is of boundary layer type. In the boundary layer the fluid decelerates from its free stream velocity to zero velocity at the centre line, but since the density of the particle material is much greater than the fluid density, the particles cannot accommodate this rapid deceleration but tend to slip through the fluid as they decelerate.

Because of the particle slip velocities, there will be a volume force acting on the fluid and an equal opposite force acting on the particle phase. This volume force is assumed to have the form of the Stoke's drag law. For large particle Reynolds number, this assumption is erroneous, so it will be assumed that the particle Reynolds number is of order unity. Every where this restriction is not met, the result still will be qualitatively correct and quantitatively reasonable.

Therefore $\vec{F}_{\rm p} = n_{\rm p} 6\pi\mu a(\bar{v}_{\rm p} - \bar{v})$. Now in the boundary layer the particles find themselves in a shear flow which causes them to rotate, thus giving rise to a lift force acting on the particles in addition to the Stokes drag forces. Unfortunately the problem of the sphere in a shear flow has not been done. In two dimensions

$$\left|\frac{\vec{F}_{\rm p1}}{\vec{F}_{\rm p}}\right| = \frac{n_{\rm p}\pi a^3 \rho_2^1((\partial v/\partial x) + (\partial u/\partial y))v_{\rm p} - v}{n_{\rm p}6\pi\mu a|v_{\rm p} - v|}$$

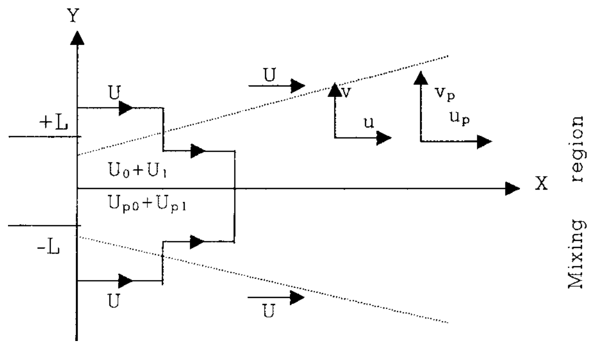

Figure 1. Schematic view of the two phases flow model.

or

$$\left|\frac{\vec{F}_{p1}}{\vec{F}_p}\right| = \frac{1}{12}\left(\frac{\rho u_\infty a}{\mu}\right)\left(\frac{a}{L}\right)\left(\frac{1}{\sqrt{R_e}}\frac{\partial v^*}{\partial x^*} + \sqrt{R_e}\frac{\partial u^*}{\partial y^*}\right)$$

where $u, v = x$ and y component of $\vec{V}$ (Figure 1).

L is some characteristic length. Also the non-dimensional quantities are $x^* = x/L$, $R_e = \rho u_\infty L/\mu$, $y^* = y/L\sqrt{R_e}$, $u^* = u/u_\infty$, $v^* = \sqrt{R_e}\, v/u_\infty$. Then if $(\rho u_\infty a/\mu)(a/L)$ is small enough, we can neglect lift force compared to Stokes drag forces.

But $(\rho u_\infty a/\mu)(a/L) = (\rho u_\infty L/\mu)(a^2/L^2) = R_e(a^2/L)$, and cannot be always small enough and lift force cannot be neglected.

Taking the x-axis to be along the axis of the jet, the y-axis perpendicular to it and the origin at the nozzle exit and at the mid-point of the nozzle width, the governing equations of two-dimensional jet flow studied by, Rhyming [6], and Datta and Mishra [7] are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$(1-\varphi)\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = v\frac{\partial^2 u}{\partial y^2} + \frac{\rho_p}{\rho}\left(\frac{u_p - u}{\tau_p}\right) \tag{2}$$

$$\varphi\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \frac{u - u_p}{\tau_p} - v_p\frac{\partial^2 u_p}{\partial y^2} \tag{3}$$

$$\varphi\left(u_p\frac{\partial v_p}{\partial x} + v_p\frac{\partial v_p}{\partial y}\right) = \frac{v - v_p}{\tau_p} - v_p\frac{\partial^2 v_p}{\partial y^2} - \frac{1}{2}\frac{\pi a^3 \rho}{m_p}(u - u_p)\left(\frac{\partial u}{\partial y}\right) \tag{4}$$

$$\frac{\partial}{\partial x}(\rho_p u) + \frac{\partial}{\partial y}(\rho_p v) = D_p\frac{\partial^2 \rho_p}{\partial y^2} \tag{5}$$

Now introducing the non-dimensional variables

$$\bar{x}=\frac{x}{L},\quad \bar{y}=\frac{y}{L},\quad \bar{u}=\frac{u}{U},\quad \bar{v}=\frac{v}{U},\quad \bar{u}_{\rm p}=\frac{u_{\rm p}}{U},\quad \bar{v}_{\rm p}=\frac{v_{\rm p}}{U},\quad \bar{\rho}_{\rm p}=\frac{\rho_{\rm p}}{\rho_{\rm p0}}$$

Equations (1–5) become

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0 \tag{6}$$

$$(1-\varphi)\left(\bar{u}\,\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\,\frac{\partial\bar{u}}{\partial\bar{y}}\right)=k_1\,\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\alpha\,\frac{\bar{\rho}_{\rm p}}{\Lambda}(\bar{u}_{\rm p}-\bar{u}) \tag{7}$$

$$\varphi\left(\bar{u}_{\rm p}\frac{\partial\bar{u}_{\rm p}}{\partial\bar{x}}+\bar{v}_{\rm p}\frac{\partial\bar{u}_{\rm p}}{\partial\bar{y}}\right)=\frac{\bar{u}-\bar{u}_{\rm p}}{\Lambda}-k_2\,\frac{\partial^2\bar{u}_{\rm p}}{\partial\bar{y}^2} \tag{8}$$

$$\varphi\left(\bar{u}_{\rm p}\,\frac{\partial\bar{v}_{\rm p}}{\partial\bar{x}}+\bar{v}_{\rm p}\,\frac{\partial\bar{v}_{\rm p}}{\partial\bar{y}}\right)=\frac{\bar{v}-\bar{v}_{\rm p}}{\Lambda}-k_2\,\frac{\partial^2\bar{v}_{\rm p}}{\partial\bar{y}^2}-\frac{\pi a^3\rho}{2m_{\rm p}}(\bar{u}-\bar{u}_{\rm p})\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right) \tag{9}$$

$$\frac{\partial}{\partial\bar{x}}(\bar{\rho}_{\rm p}\bar{u})+\frac{\partial}{\partial\bar{y}}(\bar{\rho}_{\rm p}\bar{v})=k_2\,\frac{\partial^2\bar{\rho}_{\rm p}}{\partial\bar{y}^2} \tag{10}$$

where

$$k_1=\frac{1}{R_{\rm e}},\quad k_2=\frac{1}{R_{\rm e_p}},\quad R_{\rm e_p}=\frac{UL}{v_{\rm p}}$$

Considering the flow from the orifice under full expansion, we can assume that, (i) the pressure in the mixing region to be approximately constant and the pressure at the exit of the nozzle to be equal to that of the surrounding stream, and (ii) the velocity components in the jet is only slightly different from that of the surrounding stream. Hence after dropping bars we can write, $u=u_0+u_1,\ v=v_1,\ u_{\rm p}=u_{\rm p0}+u_{\rm p1},\ v_{\rm p}=v_{\rm p1},\ \rho_{\rm p}=\rho_{\rm p1}$. Since the particle slips, $u_0,\ u_{\rm p0}$ should not be equal, $u_1,u_{\rm p1},v_1,v_{\rm p1},\rho_{\rm p1}$, are here perturbation quantities and $u_1\ll u_0, u_{\rm p1}\ll u_{\rm p0},\ u_1\approx v_1,\ u_{\rm p1}\approx v_{\rm p1}$. Particle concentration is throughout taken as a perturbation quantity only because the bulk concentration of particles in the ensuing fluid is taken to be small.

From the above consideration, Equations (6)–(10) can be written in the non-dimensional linearized form, after dropping the suffix 1, as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{11}$$

$$(1-\varphi)u_0\,\frac{\partial u}{\partial x}=k_1\frac{\partial^2 u}{\partial y^2}+\frac{\alpha\rho_{\rm p}}{\Lambda}(u_{\rm p0}-u_0) \tag{12}$$

$$\varphi u_{\rm p0}\frac{\partial u_{\rm p}}{\partial x}=\frac{u_0-u_{\rm p0}}{\Lambda}-k_2\frac{\partial^2 u_{\rm p}}{\partial y^2}+\frac{(u-u_{\rm p})}{\Lambda} \tag{13}$$

$$\varphi u_{p0}\frac{\partial v_p}{\partial x} = \frac{(v - v_p)}{\Lambda} - k_2\frac{\partial^2 v_p}{\partial y^2} - \beta(u_0 - u_{p0})\frac{\partial u}{\partial y} \tag{14}$$

$$u_0\frac{\partial \rho_p}{\partial x} = k_2\frac{\partial^2 \rho_p}{\partial y^2} \tag{15}$$

where $\beta = \pi a^3\rho/2m_p$ is the lift force parameter.

Equations (11)–(15) have to be solved for the following entry and boundary conditions:

$$u_y(x,0) = 0, \quad u(x,\infty) = 0 \tag{16}$$

$$u_{py}(x,0) = 0, \quad u_p(x,\infty) = 0 \tag{17}$$

$$\rho_{py}(x,0) = 0, \quad \rho_p(x,\infty) = 0 \tag{18}$$

$$\left.\begin{aligned} v(0,y) &= 0, \quad v(x,0) = 0 \\ v_p(0,y) &= 0, \quad v_p(x,0) = 0 \end{aligned}\right\} \tag{19}$$

$$\frac{\partial u_p}{\partial y}(x,\infty) = 0 \tag{20}$$

Now putting $\xi = k_1 x$, the governing Equations (11)–(15) become

$$\frac{\partial u}{\partial \xi} + \frac{1}{k_1}\frac{\partial v}{\partial y} = 0 \tag{21}$$

$$(1-\varphi)u_0\frac{\partial u}{\partial \xi} = \frac{\partial^2 u}{\partial y^2} + \alpha k\rho_p(u_{p0} - u_0) \tag{22}$$

$$\varphi u_{p0}\frac{\partial u_p}{\partial \xi} = k(u_0 - u_{p0}) - \varepsilon\frac{\partial^2 u_p}{\partial y^2} + k(u - u_p) \tag{23}$$

$$\varphi u_{p0}\frac{\partial v_p}{\partial \xi} = k(v - v_p) - \varepsilon\frac{\partial^2 v_p}{\partial y^2} - \frac{\beta}{k_1}(u_0 - u_{p0})\frac{\partial u}{\partial y} \tag{24}$$

$$u_0\frac{\partial \rho_p}{\partial \xi} = \varepsilon\frac{\partial^2 \rho_p}{\partial y^2} \tag{25}$$

where $\varepsilon = k_2/k_1$, $k = 1/\Lambda k_1$.

3. METHOD OF SOLUTION

Equations (21)–(25) have been solved by the finite difference method using implicit schemes, of the Crank–Nicholson and the discretized version of the PDE (21)–(25) can be presented as

$$\begin{aligned} -A_i\rho_{\mathrm{p}i-1,n+1} + B_i\rho_{\mathrm{p}i,n+1} - C_i\rho_{\mathrm{p}i+1,n+1} &= D_i \\ -A_iu_{i-1,n+1} + Q_iu_{i,n+1} - C_iu_{i+1,n+1} &= S_i \\ -A_iu_{\mathrm{p}i-1,n+1} + Q_{\mathrm{p}i}u_{\mathrm{p}i,n+1} - C_iu_{\mathrm{p}i+1,n+1} &= S_{\mathrm{p}i} \\ -A_iv_{\mathrm{p}i-1,n+1} + R_iv_{\mathrm{p}i,n+1} - C_iv_{\mathrm{p},n+1} &= H_i \end{aligned} \tag{26}$$

where $A_i = 1,\ C_i = 1,\ \lambda = \Delta\xi/\Delta y^2, B_i = 2 + (2\Delta y^2/\varepsilon\,\Delta\xi)u_0$

$$D_i = \rho_{\mathrm{p}i-1,n} + \left(-2 + \frac{2\Delta y^2 u_0}{\varepsilon\,\Delta\xi}\right)\rho_{\mathrm{p}i,n} + \rho_{\mathrm{p}i+1,n}$$

$$Q_i = 2 + \frac{2\Delta y^2(1-\varphi)}{\Delta\xi}u_0$$

$$S_i = u_{i-1,n} + \left(-2 + \frac{2\Delta y^2(1-\varphi)}{\Delta\xi}u_0\right)u_{i,n} + u_{i+1,n} + 2\Delta y^2\alpha k\rho_{\mathrm{p}}(u_{\mathrm{p}0} - u_0)$$

$$Q_{\mathrm{p}i} = 2 - \frac{2\Delta y^2\varphi u_{\mathrm{p}0}}{\varepsilon\,\Delta\xi}$$

$$\begin{aligned} S_{\mathrm{p}i} = u_{\mathrm{p}i-1,n} &+ \left(-2 - \frac{2\Delta y^2\varphi u_{\mathrm{p}0}}{\varepsilon\,\Delta\xi} + \frac{2\Delta y^2 k}{\varepsilon}\right)u_{\mathrm{p}i,n} + u_{\mathrm{p}i+1,n} \\ &- \frac{2\Delta y^2 k}{\varepsilon}u_{i,n} - \frac{2\Delta y^2 k}{\varepsilon}(u_0 - u_{\mathrm{p}0}) \end{aligned}$$

$$R_i = 2 - \frac{2\varphi\Delta y^2 u_{\mathrm{p}0}}{\varepsilon\,\Delta\xi}$$

$$\begin{aligned} H_i = v_{\mathrm{p}i-1,n} &+ \left(-2 + \frac{2\Delta y^2 k}{\varepsilon} - \frac{2\varphi\Delta y^2 u_{\mathrm{p}0}}{\varepsilon\,\Delta\xi}\right)v_{\mathrm{p}i,n} + v_{\mathrm{p}i+1,n} \\ &- \frac{2\Delta y^2 k}{\varepsilon}v_{i,n} + \frac{2\Delta y}{\varepsilon k}\beta(u_0 - u_{\mathrm{p}0})(u_{i,n} - u_{i-1,n}) \end{aligned}$$

$$\beta = \frac{1}{2}\,\frac{\pi a^3\rho}{m_{\mathrm{p}}}$$

The set of linear Difference equations is solved with prescribed boundary condition (16)–(20) giving the perturbed variables $u, u_{\mathrm{p}}, v, v_{\mathrm{p}}$, and ρ_{p} at $(x + \Delta x)$. Equations (26) are solved recursively such that the successive values of $u, u_{\mathrm{p}}, v, v_{\mathrm{p}}$, and ρ_{p} are less than 10^{-4}. With improved values of the dependent variables, Equations (26) are integrated again. The process is repeated till the difference in the values of the dependent variables in two consecutive operations are less than 10^{-4} and then the integration proceeds to next step. In downstream of the mixing region, when the difference in $u, u_{\mathrm{p}}, v, v_{\mathrm{p}}$, and ρ_{p} at Nth and $(N-1)$th grid points exceed certain prescribed value, the range of integration in y-direction is increased so

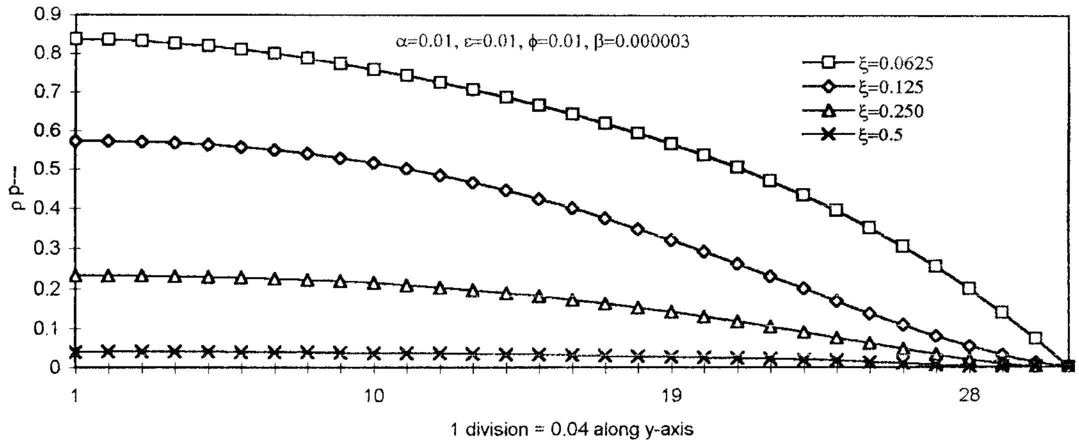

Figure 2. Density profile for the particle against $y, k = 1.0$.

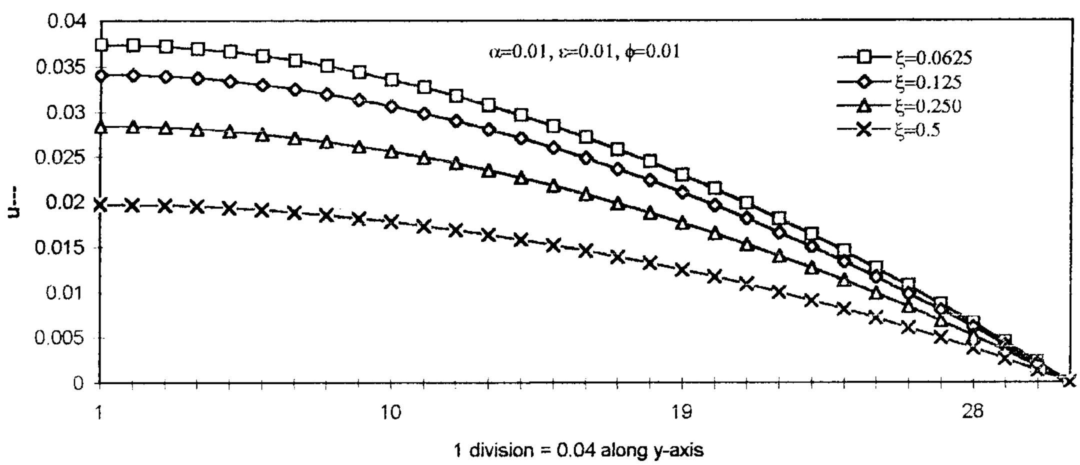

Figure 3. Perturbation of magnitude for the fluid velocity component u on the jet axis for $k = 1.0$, $\beta = 0.000003$, against y.

as to get smooth variation of the dependent variables. This procedure guarantees convergence, compatibility and stability criteria of the numerical method used.

4. DISCUSSION OF THE RESULT AND CONCLUSION

Some examples of typical results obtained in this study are exhibited in Figures 2–13. The characteristic features of gas–particulate flow in the boundary layer are presented. The following values of the parameters are used basing on the assumptions made in the formulation

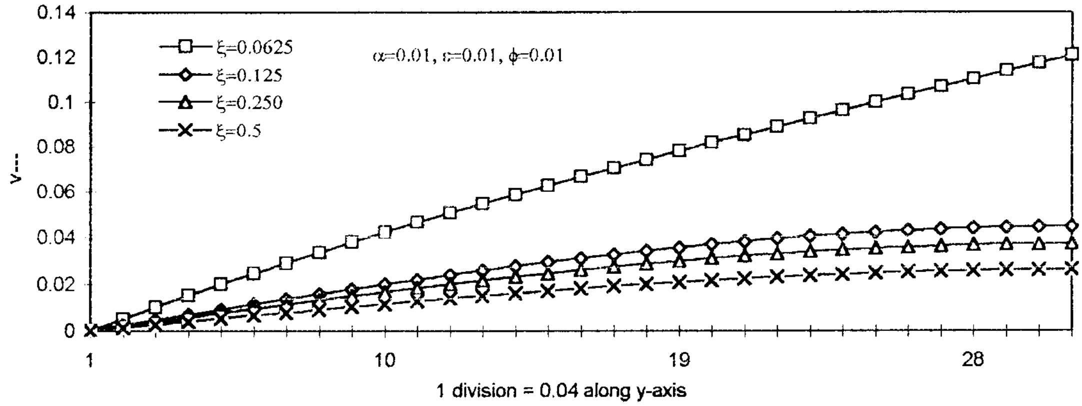

Figure 4. Perturbation of magnitude for the fluid velocity component v on the jet axis for $k=1.0$, $\beta=0.000003$, against y.

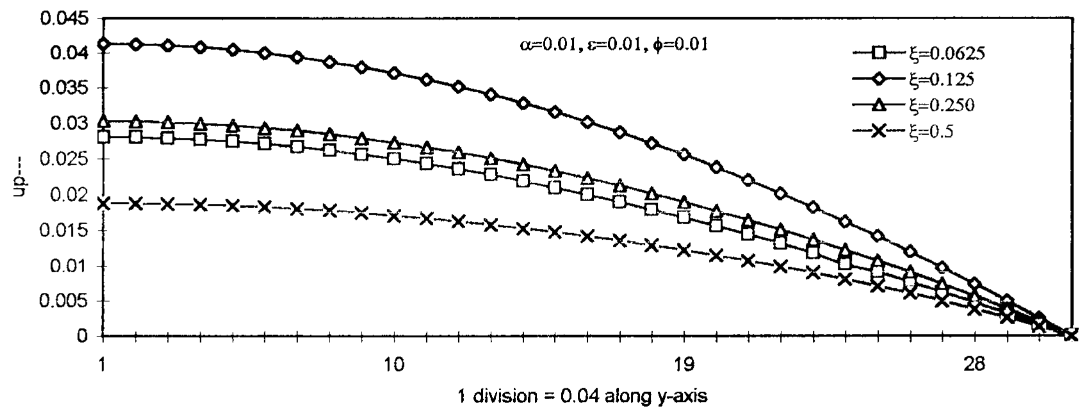

Figure 5. Perturbation of magnitude for the particle velocity component u_{p} on the jet axis for $k=1.0$, $\beta=0.000003$, against y.

of the present analysis:

$u = 60.96$ m/s
$\rho = 0.9752$ kg/m^3
$\mu = 1.5415\times10^{-6}$ kg/m^3
$\rho_{\pi\mathrm{p}} = 801.0, 1602.0, 2403.0, 8010.0$ kg/m^3
$\alpha = 0.01, 0.02, 0.03, 0.1$
$d = 50\mu, 100\mu, 250\mu$
$L = 0.3048$ m

The u-distribution depends on the parameters $\alpha, \varepsilon, \varphi, \beta$ and k. The relative influence of these parameters on u as a function of y for certain fixed ξ-values have been demonstrated in Figures 3 and 10. The ξ-values have been chosen in such a manner that if should be possible

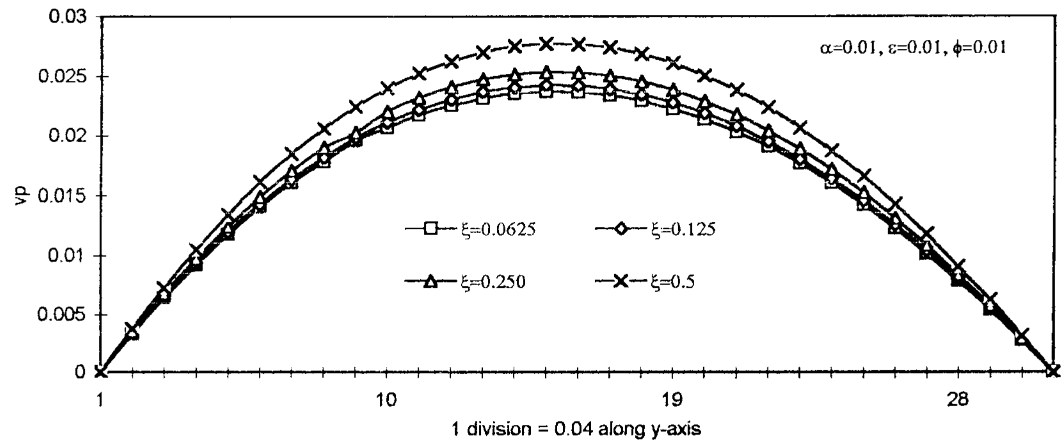

Figure 6. Perturbation of magnitude for the particle velocity component v_p on the jet axis for $k = 1.0$, $\beta = 0.000003$, against y.

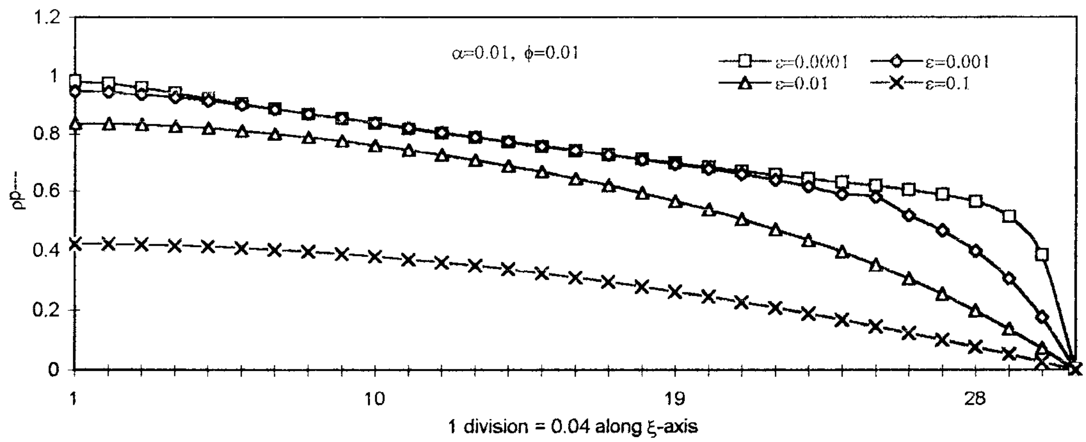

Figure 7. Density profile for the particle component ρ_p against ξ, $k = 1.0$.

to observe the influence of the above parameters on u. Hence in Figure 10 $\alpha, \varepsilon, \varphi, \beta$, and k are held constant and given the arbitrary values $\alpha = 0.01$, $\varepsilon = 0.01$, $\beta = 0.000003$ and $k = 1$, respectively, where as φ gets varied in a series of values 0.001, 0.01, 0.04, 0.08, and 0.10. For the whole series of non-dimensional quantities $u_0 = 1.0$, $u_{p0} = 0.95$, it is observed that the variation in the perturbation magnitude for the fluid velocity component u is not significant. In Figure 3, the perturbation magnitude for the fluid velocity component u on the jet axis decreases towards the downstream of the mixing region.

The Figures 5, 8 and 12 depict the shape and development of perturbation particle velocity component u_p. The u_p distribution contains five parameters $\alpha, \varepsilon, \varphi, \beta$ and k. From Figure 5, it can be observed that the magnitude of perturbation particle velocity component increases initially then starts decreasing towards the trailing edge of the mixing region. From Figures 8

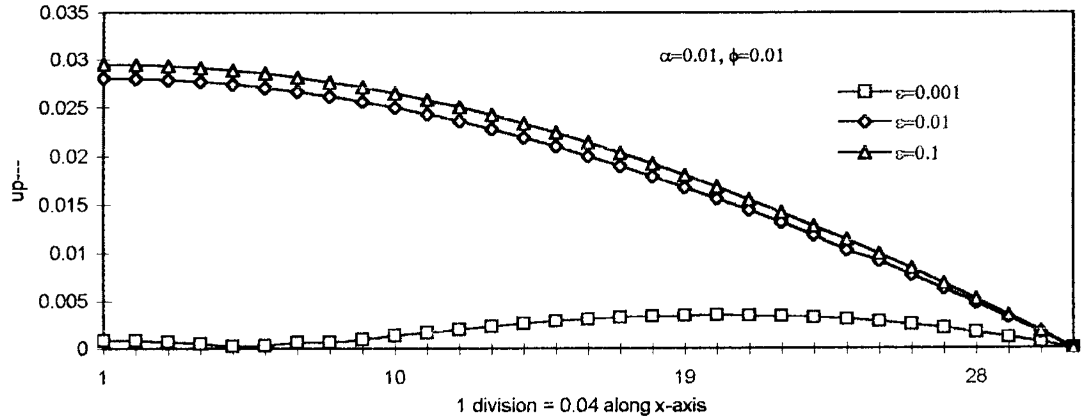

Figure 8. Perturbation of magnitude for the particle velocity component u_p on the jet axis for $k = 1.0$, $\beta = 0.000003$, against y.

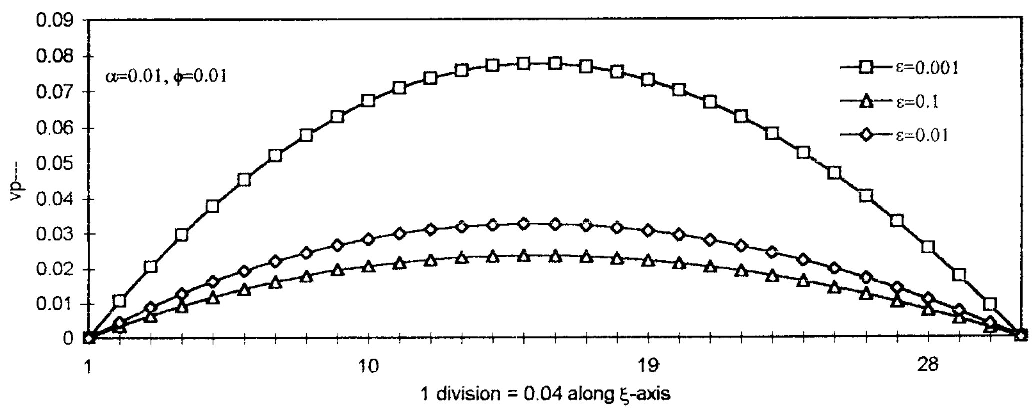

Figure 9. Perturbation of magnitude for the particle velocity component v_p on the jet axis for $k = 1.0$, $\beta = 0.000003$, against ξ.

and 12 it is observed that both ε and φ have profound influence in the shape and development of the u_p. As ε increases, the magnitude of u_p increases and as φ increases u_p decreases initially, then increases with φ. For $\varphi = 0.1$ the particles overshoot the local carrier jet speed.

The density distribution ρ_p in the mixing region can be visualized from Figures 2 and 7. From Figure 2 it is observed that the distribution of particles becomes thin towards the downstream of the mixing region. The increase of ε helps in rapid migration of the particles, thereby thinning the concentration. The fact that the increase of u_p with the increase of ε and φ supports the above physical phenomenon.

The incorporation of diffusion and volume fraction in the constitutive equation helps in rapid migration of the particles resulting in thinning of the pollutant concentration. No co-ordination and consulted efforts have been done so far by previous authors and no sufficient literature is documented to that effect.

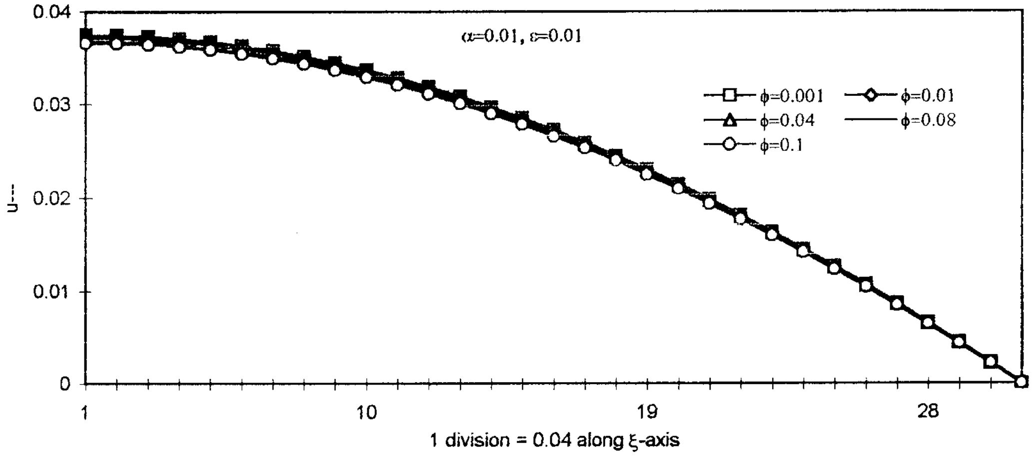

Figure 10. Perturbation of magnitude for the fluid velocity component u on the jet axis for $k=1.0$, $\beta=0.000003$, against ξ.

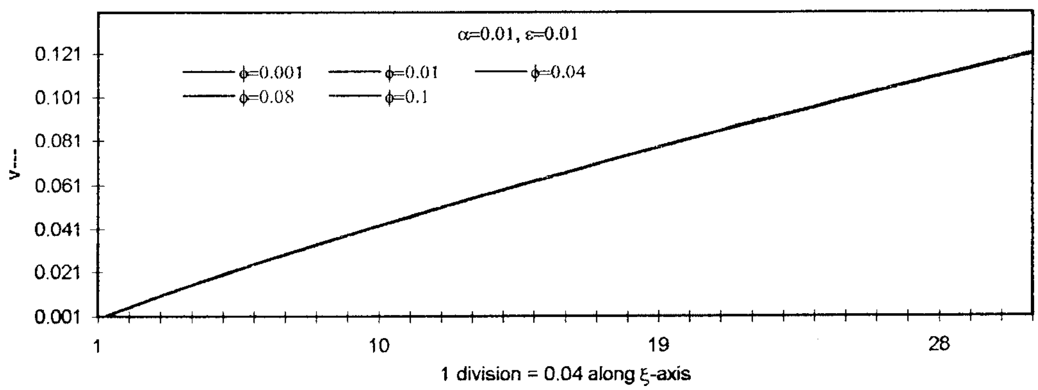

Figure 11. Perturbation of magnitude for the fluid velocity component v on the jet axis for $k=1.0$, $\beta=0.000003$, against ξ.

NOMENCLATURE

a	radius of each particle
D_p	binary diffusion coefficient
$\vec{F}_p$	Stokes drag force, $\vec{F}_p = n_p 6\pi\mu a(v - v_p)$
$\vec{F}_{p1}$	Lift force, $\vec{F}_p = n_p \pi a^3 \rho \dfrac{(\nabla \times \vec{V})}{2} \times (\vec{V}_p - \vec{V})$
L	half-width of the jet
K_1	inverse of fluid Reynolds number $(=1/R_e = v/UL)$
K_2	inverse of particle Reynolds number (v_p/UL)
N_p	mass of the particle

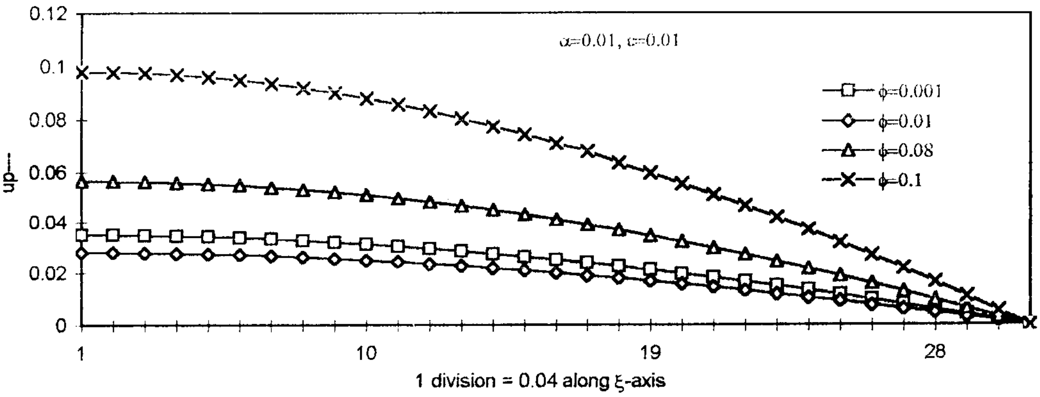

Figure 12. Perturbation of magnitude for the particle velocity component u_p on the jet axis for $k = 1.0$, $\beta = 0.000003$, against ξ.

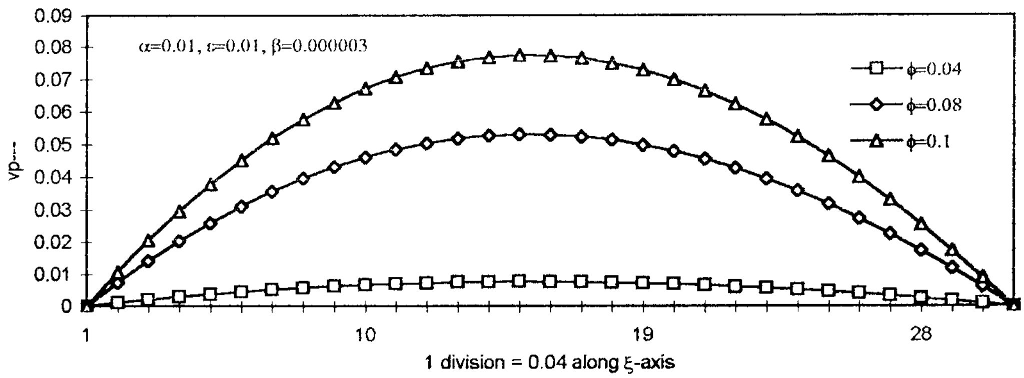

Figure 13. Perturbation of magnitude for the particle velocity component v_p on the jet axis for $k = 1.0$, $\beta = 0.000003$, against ξ.

n_p	number of particles per unit volume of the mixture
U	free stream velocity of clear fluid
(u^0, u_{p0})	the fluid and particle velocity at the exit of the nozzle
$(\bar{u}, \bar{v})$, (u_p, v_p)	non-dimensional velocity components of fluid and particle phase, respectively
$\vec{v}(u, v), \vec{v}_p(u_p, v_p)$	fluid and particle phase velocities
(x, y)	space co-ordinates along and the axis of the jet perpendicular to it
$(\bar{x}, \bar{y})$	dimensionless co-ordinates
α	concentration parameter $= (\rho_p/p)$
β	lift force parameter $= \pi a^3 \rho/2m_p$
ε	diffusion parameter $= (v_p/v)v_p \approx D_p$
Λ	non-dimensional length $(= \lambda/L)$
(ν, ν_p)	kinematics coefficient of viscosity of fluid and particle phase respectively

(ρ, ρ_p)	density of fluid and particle phase, respectively
ρ_s	mass of particle material per unit volume of particle material
ρ_{p0}	initial particle mass concentration in the jet
$\tau_p = (2/9)(\rho_s a/\mu)$	the momentum equilibration time
μ	coefficient of viscosity of fluid
φ	volume fraction of dust particles
ρ_p	density of the particles in the free-stream
λ	momentum relaxation length $(= \tau_p U)$
ξ	modified x-co-ordinate $(= K_1 x)$

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